Stewart's Theorem

Let $a$, $b$ and $c$ be the lengths of the sides of a triangle $ABC$. Consider any point $P$ on $AB$ and the following notation: $AP=m$ and $BP=n$. Then the Stewart's theorem (also called Apollonius’ theorem), states that $ma^2+nb^2=c(PC^2+mn)$.

Proof:

The law of cosines for $\Delta PAC$ and $\Delta PBC$ states:

\[ b^2 = m^2 + PC^2 - 2mPC\cos(\angle PA, PC) \quad (1) \]
\[ a^2 = n^2 + PC^2 - 2nPC\cos(\angle PB, PC) = n^2 + PC^2 + 2nPC\cos(\angle PA, PC) \quad (2) \]

We used that $\cos(\angle PA, PC) = -\cos(\angle PB, PC)$ due to the fact that angle $(PA, PC)$ and angle $(PB, PC)$ are supplementary.

Multiplying the equation (1) by $n$, the equation (2) by $m$, and add to eliminate $\cos(\angle PA, PC)$, we obtain

\[ mb^2 + na^2 = nm^2 + n^2m + (m+n)PC^2 = (m+n)(nm + PC^2) = c(nm + PC^2), \]